WAVE CELERITY IN THE INNER SURF ZONE

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In this paper, the ability of the Saint Venant shock-wave theory for predicting broken-wave kinematics in the inner surf zone is discussed. We show that this approach is based on hypotheses which are less restrictive than those of the classical bore model. We derive a new analytical expression from the shock-wave theory to improve regular broken-wave celerity prediction. Model results fit better celerity measurements than the classical bore model. The new expression represents a useful alternative when evaluating broken-wave celerity in time-averaged wave models.

1. Introduction

Quantitative predictions of wave propagation in the nearshore require accurate modelling of wave celerity. Although well-stated theories have been developed to describe shoaling zone kinematics, much remains to be done for the surf zone, where broken-wave celerity c_b constitutes a key parameter for wave models.

For time-averaged wave models, volume flux, energy flux and energy dissipation are functions of c_b. In these models, celerity is generally estimated by using either the linear shallow water theory, or the classical non-linear bore model (Svendsen et al., 1978, 2003).

For time-dependent Boussinesq-type models, which are usually based on the roller concept (Schäffer et al., 1993; Sorensen et al., 1998), an estimate of c_b is required to compute roller velocity. Generally, a rough estimate, $c_b = 1.3(gd)^{1/2}$ (d is the still water depth), is used.

Then, if we are to improve wave modelling it is essential to establish an accurate representation of broken-wave celerity.

Initial numerical-based studies by Kobayashi et al. (1989) and Cox et al. (1994), completed by recent theoretical studies (Bonneton, 2001, 2004) have shown that the Saint Venant equations, combined with the shock-wave concept, provide an appropriate theoretical framework to determine wave transformation in the inner surf zone (ISZ).

The present paper focuses on the ability of this shock-wave model to predict ISZ broken-wave celerity c_b. After a review of the classical bore model in section 2, we introduce the shock-wave model in section 3. A new analytical expression for c_b, based on this model, is presented in section 4; and final conclusions are drawn in section 5.

2. The classical bore model

The simple linear shallow water theory fails to predict the actual broken-wave celerity because of the non-linear behaviour of broken-waves in the surf zone. Svendsen et al. (1978) developed a non-linear c_b-model based on the classical analogy, introduced by Le Méhauté (1962)), between a breaking wave and an hydraulic jump (see figure 1). A recent review of this model is given by Svendsen et al. (2003).

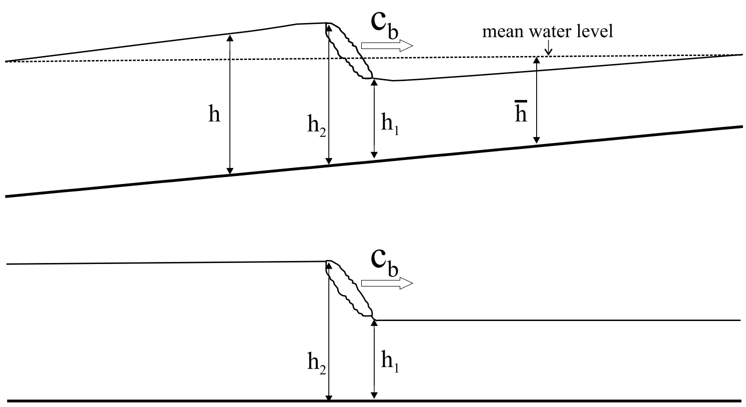

Figure 1. Classical analogy between broken-wave and hydraulic jump. h is the water depth and subscripts 1 and 2 respectively indicate values ahead and behind the wave front.

This model relies on two sets of approximations. The first one corresponds to the Saint Venant hypotheses:

- A1: both vertical non-uniformity of the horizontal velocity and non-hydrostatic effects are negligible

The second set of approximations stems from the classical analogy between broken waves and hydraulic jumps, illustrated in figure 1:

- A2: the bottom is considered as a locally horizontal bottom
- A3: breaking waves are considered as saturated breakers
- A4: both upstream and downstream flows are uniform
- A5: the wave has a quasi-constant form

From mass and momentum conservation equations and using all the previous approximations it is straightforward to determine both Q_b the volume flux across the jump in the reference frame moving at c_b ($Q_b = h_1(u_1 - c_b) = h_2(u_2 - c_b)$, where u is the fluid velocity, h the water depth and subscripts 1 and 2 respectively indicate values ahead and behind the jump), and D_b the local dissipation. The volume flux Q_b is given by

$$Q_b = -\Big(\frac{gh_1h_2(h_2 + h_1)}{2}\Big)^{\frac{1}{2}} \tag{1}$$

and the local dissipation D_b writes

$$D_b = -\frac{\rho g}{4}\frac{Q_bH^3}{h_1h_2} = \frac{\rho g}{4}\Big(\frac{g(h_2 + h_1)}{2h_1h_2}\Big)^{\frac{1}{2}}(h_2 - h_1)^3. \tag{2}$$

For a stationary wave field the total mean volume flux, $\bar{Q} + c_b\bar{h}$ ($Q = h(u - c_b)$ is the volume flux in the reference frame moving at c_b), is equal to zero. Since we assume that the wave has a quasi-constant form (A5 approximation), $\bar{Q} = Q_b$ and then

$$Q_b = -c_b\bar{h}.$$

Combining this expression with Eq. (1) we find the classical bore model for the broken-wave celerity

$$c_b = \Big(\frac{gh_1h_2(h_1 + h_2)}{2\bar{h}^2}\Big)^{\frac{1}{2}}, \tag{3}$$

and for the mean dissipation $\bar{D}_b = D_b f / c_b$

$$\bar{D}_b = \frac{\rho g}{4}\frac{f\bar{h}H^3}{h_1h_2}, \tag{4}$$

where f is the wave frequency and H is the wave height (for saturated breakers $H = h_2 - h_1$).

Most of time-averaged wave models are based on Eq. (3) and (4). However, we have seen that these equations are associated with five strong approximations. We will show in the next section that an alternative approach to determine c_b and $\bar{D}_b$ can be developed in a less restrictive context.

3. Saint Venant shock-wave model

We can distinguish two regions in ISZ broken-waves (see figure 2): a thin wave front where the flow variables change rapidly, and a regular wave region. Bonneton (2004) showed that approximation A1 is well satisfied in the regular wave region and that wave fronts can be approximated by introducing discontinuities (see figure 2) satisfying appropriate shock conditions.

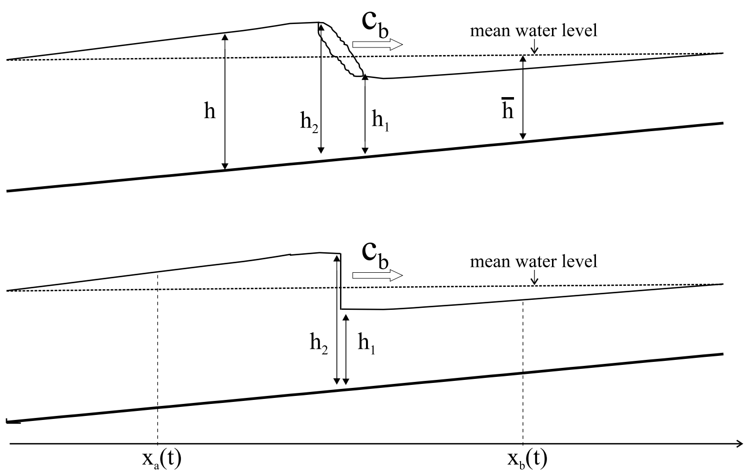

Figure 2. Broken-wave and shock-wave. h is the water depth and subscripts 1 and 2 respectively indicate values ahead and behind the wave front.

To derive these shock conditions, the laws of conservation of mass and momentum are applied to the fluid domain $[x_a, x_b]$:

$$\frac{d}{dt}\left(\int_{x_a(t)}^{x_b(t)} \rho h \, dx\right) = 0$$

$$\frac{d}{dt}\left(\int_{x_a(t)}^{x_b(t)} \rho h u \, dx\right) = F_o$$

where F_o is the x-component of the sum of body and surface forces acting on the fluid domain. Bonneton (2004) extends the classical demonstration of Saint Venant shock-wave solutions (Stoker, 1957) by taking into account non-flat bottom and friction effects. We consider the limit case in which the length of the domain tends to zero. In continuous part of the flow these conservation equations reduce to the Saint Venant equations

$$\frac{\partial \zeta}{\partial t} + \frac{\partial hu}{\partial x} = 0 \quad (5)$$

$$\rho\frac{\partial hu}{\partial t} + \rho\frac{\partial}{\partial x}\left(hu^2 + \frac{1}{2}gh^2\right) = \rho gh\frac{\partial d}{\partial x} - \tau_b \ , \quad (6)$$

where τ_b is the bottom-shear stress. At the discontinuity, conservation equations reduce to the following shock conditions

$$u_1 - c_b = -\left(\frac{gh_2}{2h_1}(h_2 + h_1)\right)^{\frac{1}{2}} \quad (7)$$

$$u_2 - c_b = -\left(\frac{gh_1}{2h_2}(h_2 + h_1)\right)^{\frac{1}{2}} \ . \quad (8)$$

These conditions are equivalent to Eq. (1) for the classical bore model, but have been obtained without the four strong approximations A2-5. In particular, we do not need to assume a locally flat bottom or a quasi-constant wave form. We can also consider non-saturated breakers which correspond to shock height $h_2 - h_1$ smaller than the wave height H.

The mean energy dissipation is given by

$$\bar{D}_b = \frac{\rho g}{4}\frac{f}{c_b}\left(\frac{g(h_2 + h_1)}{2h_1h_2}\right)^{\frac{1}{2}}(h_2 - h_1)^3 \ , \quad (9)$$

which is a function of c_b. It is worthwhile to note that this expression is more general than Eq. (4), and reduces to this equation when c_b is estimated with the classical bore celerity given by Eq. (3).

The time-dependent shock-wave model, based on equations (5)-(6) and shock conditions (7) and (8), gives very good predictions of the broken-wave celerity for both regular waves (Cox, 1995; Bonneton, 2004) and irregular waves (Bonneton and Dupuis, 2000; Bonneton et al., 2004). Shock conditions could be also useful to estimate the roller velocity in time-dependent Boussinesq-type models.

4. A one-way celerity model

For time-averaged wave model, an analytical c_b-expression, like Eq. (3), is required. To obtain such an expression from the Saint Venant shock-wave model, we used an approximation less restrictive than the classical bore model approximation A5. Bonneton (2001, 2004) has shown that for regular wave propagating in the ISZ on a low-slope beach, reflexion is negligible and that Riemann invariant $u - 2(gh)^{1/2}$ can be estimated by the following relation: $u - 2(gh)^{1/2} \simeq \bar{u} - 2(g\bar{h})^{1/2}$. Neglecting the small

$\bar{u}$-contribution and combining this expression with Eq. (7) we obtain a new one-way celerity model

$$c_b = -2(g\bar{h})^{1/2} + 2(gh_1)^{1/2} + \left(\frac{gh_2}{2h_1}(h_2 + h_1)\right)^{\frac{1}{2}} . \qquad (10)$$

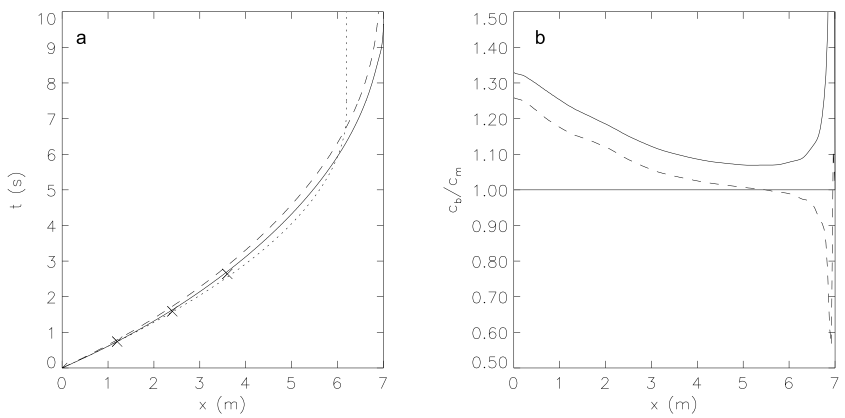

Figure 3. Comparisons between c_b-models and laboratory experiments by Cox (1995). a, broken-wave trajectories; b, c_b/c_m with $c_m = (g\bar{h})^{1/2}$. One-way celerity model Eq. (10) (solid line); classical bore model Eq. (1) (long-dashed line); roller velocity parameterization Eq. (11) (short-dashed line); experimental data (×).

In order to test this c_b-law we compare its solutions to laboratory measurements of regular waves propagating on low-slope planar beaches. These data sets include spilling breaking measurements (Cox (1995), Hansen and Svendsen (1979) and Stive (1984) (test1)) and plunging breaking measurements (Stive (1984) (test2)). In addition, Eq. (10)-solutions are compared with the classical bore celerity model Eq. (1) and the roller velocity parameterization used in Boussinesq-type models:

$$c_b = 1.3(gd)^{1/2} . \qquad (11)$$

We present in figure 3 a comparison between experimental wave front positions (Cox, 1995) and computed wave front trajectories. The whole wave field is determined with the complete Saint Venant shock-wave model (Bonneton, 2004), then the trajectories are computed with the c_b-laws given by equations (10), (1) and (11). Figure 3a shows a very good agreement between the experimental wave front positions and the trajectory computed

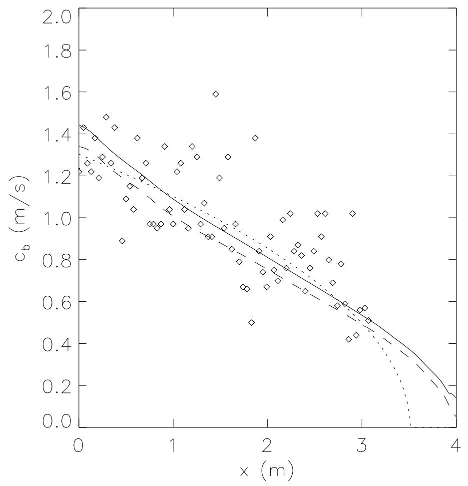

Figure 4. Comparisons between c_b-models and laboratory experiments by Hansen and Svendsen (1979). One-way celerity model Eq. (10) (solid line); classical bore model Eq. (1) (long-dashed line); roller velocity parameterization Eq. (11) (short-dashed line); experimental data (◇).

with Eq. (10). We observe that trajectories computed from Eq. (1) and Eq. (11) are close to the measured trajectories. However, the expression (11) slightly overestimates the wave front celerity and fails in the swash zone; and Eq. (1) slightly underestimates the wave front celerity (see also figure 3b). Figure 3b shows that in the ISZ c_b is greater than the linear shallow water celerity $(g\bar{h})^{1/2}$ and, as already noticed by Svendsen et al. (2003), the ratio $c_b/(g\bar{h})^{1/2}$decreases shoreward.

Figure 4 presents direct broken-wave celerity measurements performed by Hansen and Svendsen (1979). In spite of significant scatter in the measured results, we can see that the one-way celerity model gives a good prediction for the spatial variation of c_b.

Comparisons with direct measurements of the broken-wave celerity performed by Stive (1984) are presented in figures 5 (spilling breaking) and 6 (plunging breaking). In both cases, the one-way model gives better results than both the classical bore model and the roller velocity parameterization. Figures 5b and 6b show a shoreward decrease of the ratio $c_b/(g\bar{h})^{1/2}$, which is well predicted by the one-way celerity model.

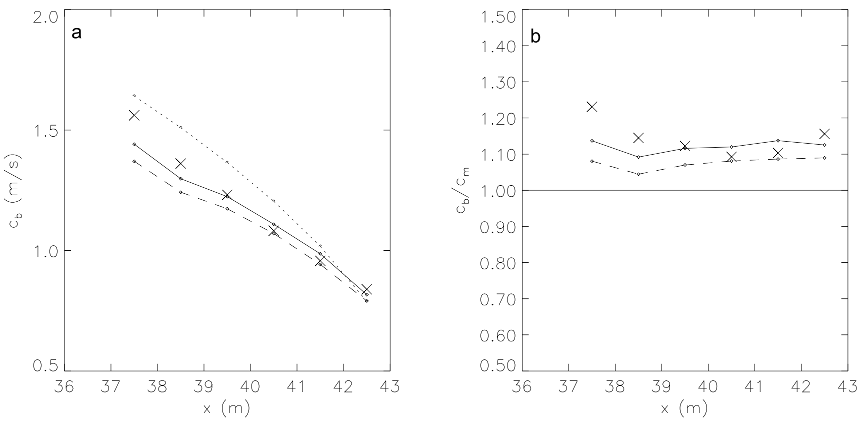

Figure 5. Comparisons between c_b-models and laboratory experiments by Stive (1984) (test1 data: spilling breaking). One-way celerity model Eq. (10) (solid line); classical bore model Eq. (1) (long-dashed line); roller velocity parameterization Eq. (11) (short-dashed line); experimental data (×). a, broken-wave celerity c_b; b, c_b/c_m with $c_m = (g\bar{h})^{1/2}$.

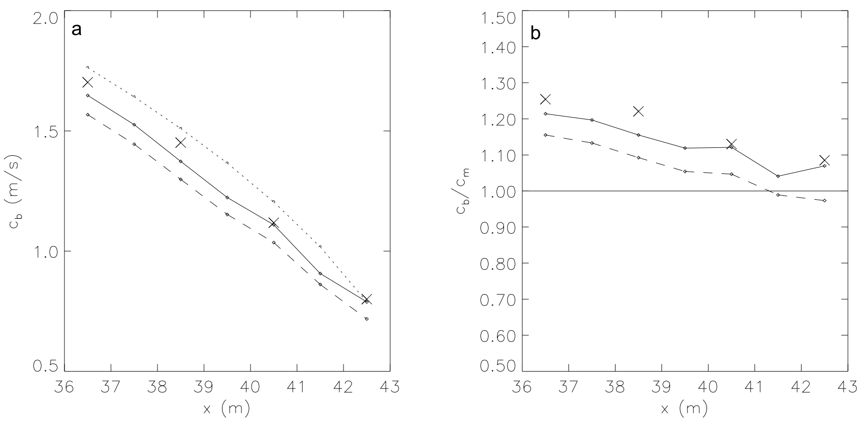

Figure 6. Comparisons between c_b-models and laboratory experiments by Stive (1984) (test2 data: plunging breaking). One-way celerity model Eq. (10) (solid line); classical bore model Eq. (1) (long-dashed line); roller velocity parameterization Eq. (11) (short-dashed line); experimental data (×). a, broken-wave celerity c_b; b, c_b/c_m with $c_m = (g\bar{h})^{1/2}$.

5. Conclusion

The ability of the Saint Venant shock-wave theory to predict broken-wave kinematics in the inner surf zone is discussed. We show that this approach is based on hypotheses which are less restrictive than those of the classical bore model. Based on the shock-wave theory a new analytical one-way model for predicting regular broken-wave celerity c_b is presented. Comparisons with the few available c_b-measurements show that this model gives better results than both the classical bore model and the roller velocity parameterization used in Boussinesq-type methods. These results indicate that for time-averaged wave models this new celerity formulation can represent an useful alternative to the classical bore model.

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